Eigenvalues In Riemannian Geometry

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Riemannian Geometry American Mathematical Soc.
In this thesis Sturm Liouville problems are studied in relation to eigenvalue problems in Riemannian geometry and some standard comparison theorems for eigenvalues are proven in the case of spherically symmetric domains in warped products. The author's main goal is to investigate fourth order Sturm Liouville operators and the Bilaplacian. The eigenfunctions of the clamped plate problem on discs are characterized, and a generalization of Szego's lower bound of the first eigenvalue to positively curved warped products is proven.
Dirac Operators in Riemannian Geometry Springer Verlag
This cutting-edge, standard-setting text explores the spectral geometry of Riemannian submersions. Working for the most part with the form valued Laplacian in the class of smooth compact manifolds without boundary, the authors study the relationship-if any-between the spectrum of Dp on Y and Dp on Z, given that Dp is the p form valued Laplacian and pi: Z ® Y is a Riemannian submersion. After providing the necessary background, including basic differential geometry and a discussion of Laplace type operators, the authors address rigidity theorems. They establish conditions that ensure that the pull back of every eigenform on Y is an eigenform on Z so the eigenvalues do not change, then show that if a single eigensection is preserved, the eigenvalues do not change for the scalar or Bochner Laplacians. For the form valued Laplacian, they show that if an eigenform is preserved, then the corresponding eigenvalue can only increase. They generalize these results to the complex setting as well. However, the spinor setting is quite different. For a manifold with non-trivial boundary and imposed Neumann boundary conditions, the result is surprising-the eigenvalues can change. Although this is a relatively rare phenomenon, the authors give examples-a circle bundle or, more generally, a principal bundle with structure group G where the first cohomology group H1(G;R) is non trivial. They show similar results in the complex setting, show that eigenvalues can decrease in the spinor setting, and offer a list of unsolved problems in this area. Moving to some related topics involving questions of positive curvature, for the first time in mathematical literature the authors establish a link between the spectral geometry of Riemannian submersions and the Gromov-Lawson conjecture. Spectral Geometry, Riemannian Submersions, and the Gromov-Lawson Conjecture addresses a hot research area and promises to set a standard for the field. Researchers and applied mathematicians interested in mathematical physics and relativity will find this work both fascinating and important.
Riemannian Geometry Oxford University Press
This book, which focuses on the study of curvature, is an introduction to various aspects of pseudo-Riemannian geometry. We shall use Walker manifolds (pseudo-Riemannian manifolds which admit a non-trivial parallel null plane field) to exemplify some of the main differences between the geometry of Riemannian manifolds and the geometry of pseudo-Riemannian manifolds and thereby illustrate phenomena in pseudo-Riemannian geometry that are quite different from those which occur in Riemannian geometry, i.e. for indefinite as opposed to positive definite metrics. Indefinite metrics are important in many diverse physical contexts: classical cosmological models (general relativity) and string theory to name but two. Walker manifolds appear naturally in numerous physical settings and provide examples of extremal mathematical situations as will be discussed presently. To describe the geometry of a pseudo-Riemannian manifold, one must first understand the curvature of the manifold. We shall analyze a wide variety of curvature properties and we shall derive both geometrical and topological results. Special attention will be paid to manifolds of dimension 3 as these are quite tractable. We then pass to the 4 dimensional setting as a gateway to higher dimensions. Since the book is aimed at a very general audience (and in particular to an advanced undergraduate or to a beginning graduate student), no more than a basic course in differential geometry is required in the way of background. To keep our treatment as self-contained as possible, we shall begin with two elementary chapters that provide an introduction to basic aspects of pseudo-Riemannian geometry before beginning on our study of Walker geometry. An extensive bibliography is provided for further reading. Math subject classifications : Primary: 53B20 -- (PACS: 02.40.Hw) Secondary: 32Q15, 51F25, 51P05, 53B30, 53C50, 53C80, 58A30, 83F05, 85A04 Table of Contents: Basic Algebraic Notions / Basic Geometrical Notions / Walker Structures / Three-Dimensional Lorentzian Walker Manifolds / Four-Dimensional Walker Manifolds / The Spectral Geometry of the Curvature Tensor / Hermitian Geometry / Special Walker Manifolds
The Laplacian on a Riemannian Manifold Springer Science & Business Media
The basic goals of the book are: (i) to introduce the subject to those interested in discovering it, (ii) to coherently present a number of basic techniques and results, currently used in the subject, to those working in it, and (iii) to present some of the results that are attractive in their own right, and which lend themselves to a presentation not overburdened with technical machinery.
Riemannian Geometry and Geometric Analysis World Scientific
This book deals with Riemannian manifolds for which the nullity space of the curvature tensor has codimension two. These manifolds are “semi-symmetric spaces foliated by Euclidean leaves of codimension two” in the sense of Z I Szabó. The authors concentrate on the rich geometrical structure and explicit descriptions of these remarkable spaces. Also parallel theories are developed for manifolds of “relative conullity two”. This makes a bridge to a survey on curvature homogeneous spaces introduced by I M Singer. As an application of the main topic, interesting hypersurfaces with type number two in Euclidean space are discovered, namely those which are locally rigid or “almost rigid”. The unifying method is solving explicitly particular systems of nonlinear PDE. Contents:IntroductionDefinition of Semi-Symmetric Spaces and Early DevelopmentLocal Structure of Semi-Symmetric SpacesExplicit Treatment of Foliated Semi-Symmetric SpacesCurvature Homogeneous Semi-Symmetric SpacesAsymptotic Foliations and Algebraic RankThree-Dimensional Riemannian Manifolds of Conullity TwoAsymptotically Foliated Semi-Symmetric SpacesElliptic Semi-Symmetric SpacesComplete Foliated Semi-Symmetric SpacesApplication: Local Rigidity Problems for Hypersurfaces with Type Number Two in IR4Three-Dimensional Riemannian Manifolds of c-Conullity TwoMore about Curvature Homogeneous SpacesBiographyIndex Readership: Mathematicians and mathematical physicists. keywords:Riemannian Manifold;Curvature Homogeneous Space;Semi-Symmetric Space;Pseudo-Symmetric Space;Asymptotic Foliation;Hypersurface with Type Number Two;Gromov Conjecture;Lichnerowicz Formula;Nomizu Conjecture;Singer Number
A Panoramic View of Riemannian Geometry CRC Press
Explores the failure of analytic-hypoellipticity of two partial differential operators. The operators are sums of squares of real analytic vector fields and satisfy Hormander's condition. By reducing to an ordinary differential operator, the author shows the existence of non-linear eigenvalues, which is used to disprove analytic- hypoellipticity of the original operators. No index. Annotation copyrighted by Book News, Inc., Portland, OR
Riemannian Geometry American Mathematical Soc.
This book studies certain spaces of Riemannian metrics on both compact and non-compact manifolds. These spaces are defined by various sign-based curvature conditions, with special attention paid to positive scalar curvature and non-negative sectional curvature, though we also consider positive Ricci and non-positive sectional curvature. If we form the quotient of such a space of metrics under the action of the diffeomorphism group (or possibly a subgroup) we obtain a moduli space. Understanding the topology of both the original space of metrics and the corresponding moduli space form the central theme of this book. For example, what can be said about the connectedness or the various homotopy groups of such spaces? We explore the major results in the area, but provide sufficient background so that a

non-expert with a grounding in Riemannian geometry can access this growing area of research.
Michael Atiyah Collected Works Springer
The third of three parts comprising Volume 54, the proceedings of the Summer Research Institute on Differential Geometry, held at the University of California, Los Angeles, July 1990 (ISBN for the set is 0-8218-1493-1). Part 3 begins with an overview by R.E. Greene of some recent trends in Riemannia
Ordinary Differential Equation Methods for Eigenvalue Problems in Riemannian Geometry Elsevier
This volume is an English translation of Sakai's textbook on Riemannian Geometry which was originally written in Japanese and published in 1992. The author's intent behind the original book was to provide to advanced undergraduate and graudate students an introduction to modern Riemannian geometry that could also serve as a reference. The book begins with an explanation of the fundamental notion of Riemannian geometry. Special emphasis is placed on understandability and readability, to guide students who are new to this area. The remaining chapters deal with various topics in Riemannian geometry, with the main focus on comparison methods and their applications.
Geometric and Computational Spectral Theory Lecture Notes in Mathematics
Intended for a one year course, this volume serves as a single source, introducing students to the important techniques and theorems, while also containing enough background on advanced topics to appeal to those students wishing to specialise in Riemannian geometry. Instead of variational techniques, the author uses a unique approach, emphasising distance functions and special co-ordinate systems. He also uses standard calculus with some techniques from differential equations to provide a more elementary route. Many chapters contain material typically found in specialised texts, never before published in a single source. This is one of the few works to combine both the geometric parts of Riemannian geometry and the analytic aspects of the theory, while also presenting the most up-to-date research - including sections on convergence and compactness of families of manifolds. Thus, this book will appeal to readers with a knowledge of standard manifold theory, including such topics as tensors and Stokes theorem. Various exercises are scattered throughout the text, helping motivate readers to deepen their understanding of the subject.
Spectral Geometry American Mathematical Soc.
For a Riemannian manifold M, the geometry, topology and analysis are interrelated in ways that have become widely explored in modern mathematics. Bounds on the curvature can have significant implications for the topology of the manifold. The eigenvalues of the Laplacian are naturally linked to the geometry of the manifold. For manifolds that admit spin structures, one obtains further information from equations involving Dirac operators and spinor fields. In the case of four-manifolds, for example, one has the remarkable Seiberg-Witten invariants. In this text, Friedrich examines the Dirac operator on Riemannian manifolds, especially its connection with the underlying geometry and topology of the manifold. The presentation includes a review of Clifford algebras, spin groups and the spin representation, as well as a review of spin structures and $\textrm{spin}mathbb{C}$ structures. With this foundation established, the Dirac operator is defined and studied, with special attention to the cases of Hermitian manifolds and symmetric spaces. Then, certain analytic properties are established, including self-adjointness and the Fredholm property. An important link between the geometry and the analysis is provided by estimates for the eigenvalues of the Dirac operator in terms of the scalar curvature and the sectional curvature. Considerations of Killing spinors and solutions of the twistor equation on M lead to results about whether M is an Einstein manifold or conformally equivalent to one. Finally, in an appendix, Friedrich gives a concise introduction to the Seiberg-Witten invariants, which are a powerful tool for the study of four-manifolds. There is also an appendix reviewing principal bundles and connections. This detailed book with elegant proofs is suitable as a text for courses in advanced differential geometry and global analysis, and can serve as an introduction for further study in these areas. This edition is translated from the German edition published by Vieweg Verlag.
Riemannian Geometry Springer Science & Business Media
A co-publication of the AMS and Centre de Recherches Mathématiques The book is a collection of lecture notes and survey papers based on the mini-courses given by leading experts at the 2015 Séminaire de Mathématiques Supérieures on Geometric and Computational Spectral Theory, held from June 15–26, 2015, at the Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada. The volume covers a broad variety of topics in spectral theory, highlighting its connections to differential geometry, mathematical physics and numerical analysis, bringing together the theoretical and computational approaches to spectral theory, and emphasizing the interplay between the two.
Recent Progress in Mathematics Imperial College Press
Offering some of the topics of contemporary mathematical research, this fourth edition includes a systematic introduction to Kahler geometry and the presentation of additional techniques from geometric analysis.
Riemannian Symmetric Spaces of Rank One Springer Science & Business Media
The subject of this book is Osserman semi-Riemannian manifolds, and in particular, the Osserman conjecture in semi-Riemannian geometry. The treatment is pitched at the intermediate graduate level and requires some intermediate knowledge of differential geometry. The notation is mostly coordinate-free and the terminology is that of modern differential geometry. Known results toward the complete proof of Riemannian Osserman conjecture are given and the Osserman conjecture in Lorentzian geometry is proved completely. Counterexamples to the Osserman conjuncture in generic semi-Riemannian signature are provided and properties of semi-Riemannian Osserman manifolds are investigated.
Selected Topics in Cauchy-Riemann Geometry World Scientific
Pseudo-Riemannian geometry is an active research field not only in differential geometry but also in mathematical physics where the higher signature geometries play a role in brane theory. An essential reference tool for research mathematicians and physicists, this book also serves as a useful introduction to students entering this active and rapidly growing field. The author presents a comprehensive treatment of several aspects of pseudo-Riemannian geometry, including the spectral geometry of the curvature tensor, curvature homogeneity, and StanilovOCoTsankovOCoVidev theory."
Cas027 Springer
This book covers the topics of differential manifolds, Riemannian metrics, connections, geodesics and curvature, with special emphasis on the intrinsic features of the subject. It treats in detail classical results on the relations between curvature and topology. The book features numerous exercises with full solutions and a series of detailed examples are picked up repeatedly to illustrate each new definition or property introduced.
American Mathematical Soc.
This volume brings together lectures from an instructional meeting on spectral theory and geometry held under the auspices of the International Centre for Mathematical Sciences in Edinburgh. The contributions here come from world experts and many are much expanded versions of the lectures they gave. Together they survey the core material and go beyond to reach deeper results. For graduate students and experts alike, this book will be a highly useful resource.
Spectral Geometry, Riemannian Submersions, and the Gromov-Lawson Conjecture American Mathematical Soc.
This book introduces readers to the living topics of Riemannian Geometry and details the main results known to date. The results are stated without detailed proofs but the main ideas involved are described, affording the reader a sweeping panoramic view of almost the entirety of the field. From the reviews "The book has intrinsic value for a student as well as for an experienced geometer. Additionally, it is really a compendium in Riemannian Geometry." --MATHEMATICAL REVIEWS
Contact Manifolds in Riemannian Geometry American Mathematical Soc.
This book contains contributions by an impressive list of leading mathematicians. The articles include high-level survey and research papers exploring contemporary issues in geometric analysis, differential geometry, and several complex variables. Many of the articles will provide graduate students with a good entry point into important areas of modern research. The material is intended for researchers and graduate students interested in several complex variables and complex geometry.
Spectral Geometry Of The Laplacian: Spectral Analysis And Differential Geometry Of The Laplacian Springer Science & Business Media
This is a collection of the works of Michael Atiyah, a well-established mathematician and winner of the Fields Medal. It is thematically divided into volumes; this one discusses index theory.