Mean Field Games And Mean Field Type Control Theo

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Fog Radio Access Networks (F-RAN) Cambridge University Press
Mean-field game (MFG) systems are revolutionary models to describe complex multi-agent dynamic systems, such as competition between asset managers in finance and traffic congestion in population dynamics. They combine mean-field approximation techniques to describe the population with optimal control approaches to characterize a representative player. This thesis investigates both computation and modeling aspects of the mean-field games. We propose a computational method for nonlocal MFGs. Our approach relies on kernel-based representations of mean-field interactions and feature-space expansions, which yields a dimension reduction. Based on the monotone inclusion formulation, we further generalize the splitting method for nonlocal MFGs to solve a class of non-potential MFGs. In terms of modelings, we integrate the spatial epidemic models with mean-field control (MFC) models to control the propagation of pandemics. We also apply MFCs to study the optimal vaccine distribution strategy. Numerical experiments demonstrate that the proposed model effectively separates infected patients in a spatial domain and transports vaccines efficiently. Finally, we study an inverse MFG problem. We propose a model recovery algorithm to reconstruct the ground metrics and interactions in the running costs with some noisy observations.
Equilibria and Bifurcation Theory for Mean-Field Games SIAM
Mean field games and Mean field type control introduce new problems in Control Theory. The terminology “games” may be confusing. In fact they are control problems, in the sense that one is interested in a single decision maker, whom we can call the representative agent. However, these problems are not standard, since both the evolution of the state and the objective functional is influenced but terms which are not directly related to the state or the control of the decision maker. They are however, indirectly related to him, in the sense that they model a very large community of agents similar to the representative agent. All the agents behave similarly and impact the representative agent. However, because of the large number an aggregation effect takes place. The interesting consequence is that the impact of the community can be modeled by a mean field term, but when

this is done, the problem is reduced to a control problem.
Advances in Dynamic and Mean Field Games Springer
This book gathers the most essential results, including recent ones, on linear-quadratic optimal control problems, which represent an important aspect of stochastic control. It presents results for two-player differential games and mean-field optimal control problems in the context of finite and infinite horizon problems, and discusses a number of new and interesting issues. Further, the book identifies, for the first time, the interconnections between the existence of open-loop and closed-loop Nash equilibria, solvability of the optimality system, and solvability of the associated Riccati equation, and also explores the open-loop solvability of mean-filed linear-quadratic optimal control problems. Although the content is largely self-contained, readers should have a basic grasp of linear algebra, functional analysis and stochastic ordinary differential equations. The book is mainly intended for senior undergraduate and graduate students majoring in applied mathematics who are interested in stochastic control theory. However, it will also appeal to researchers in other related areas, such as engineering, management, finance/economics and the social sciences.
Mean Field Games with Imperfect Information John Wiley & Sons
This volume provides an introduction to the theory of Mean Field Games, suggested by J.-M. Lasry and P.-L. Lions in 2006 as a mean-field model for Nash equilibria in the strategic interaction of a large number of agents. Besides giving an accessible presentation of the main features of mean-field game theory, the volume offers an overview of recent developments which explore several important directions: from partial differential equations to stochastic analysis, from the calculus of variations to modeling and aspects related to numerical methods. Arising from the CIME Summer School "Mean Field Games" held in Cetraro in 2019, this book collects together lecture notes prepared by Y. Achdou (with M. Laurière), P. Cardaliaguet, F. Delarue, A. Porretta and F. Santambrogio. These notes will be valuable for researchers and advanced graduate students who wish to approach this theory and explore its connections with several different fields in mathematics.
Mean Field Games with State Constraints Princeton University Press
The contents of this book comprise an appropriate background to start working and doing research on mean-field-type control and game theory. To make the exposition and explanation even easier, we first study the deterministic optimal control and differential linear-quadratic games. Then, we progressively add complexity step-by-step and little-by-little to the problem settings until we finally study and analyze mean-field-type control and game problems incorporating several stochastic processes, e.g., Brownian motions, Poisson jumps, and random coefficients. We go beyond the Nash equilibrium, which provides a solution for non- cooperative games, by analyzing other game-theoretical concepts such as the Berge, Stackelberg, adversarial/robust, and co-opetitive equilibria. For the mean-field-type game analysis, we provide several numerical examples using a Matlab-based user-friendly toolbox that is available for the free use to the readers of this book. We present several engineering applications in both continuous and discrete time. Among these applications we find the following: water distribution systems, micro-grid energy storage, stirred tank reactor, mechanism design for evolutionary dynamics, multi-level building evacuation problem, and the COVID-19 propagation control. Julian Barreiro-Gomez Hamidou Tembine With such a demand from engineering audiences, this book is very timely and provides a thorough study of mean-field-type game theory. The strenuous protagonist of this book is to bridge between the theoretical findings and engineering solutions. The book introduces the basics first, and then mathematical frameworks are elaborately explained. The engineering application examples are shown in detail, and the popular learning approaches are also investigated. Those advantageous characteristics will make this book a comprehensive handbook of many engineering fields for many years, and I will buy one when it gets published. Zhu Han
Strong Solutions to Submodular Mean Field Games with Common Noise and Related McKean-Vlasov FBSDES Springer Science & Business Media
Graduate-level text surveys games of fixed duration, games of pursuit and evasion, the computation of saddle points, games of survival, games with restricted phase coordinates, and N-person games. 1971 edition.
Mean Field Game and its Applications in Wireless Networks Springer
Mean Field Games (MFG) are the infinite-population analogue of symmetric stochastic differential games with interacting players. By considering a limiting model with a continuum of players, the theory of MFG provides a more tractable representation and can effectively approximate a broad class of perfectly symmetric stochastic dynamic games. This thesis studies games with heterogeneous players, the heterogeneity being expressed either through a type parameter or through asymmetric interactions among players, and aims at understanding under which condition the MFG approximation remains valid for such games and, if it fails, to find a substitute model. In many real-life settings, players do not view themselves as exchangeable and accurate models should incorporate this heterogeneity. We first adapt the MFG paradigm to model more heterogeneous agents by introducing a type parameter in a financial problem that has gained huge interest in the recent years: the competitive Merton problem under relative performance criteria.
Mean Field Games and Mean Field Type Control Theory CRC Press
The aim of this Thesis is to study deterministic mean field games with state constraints. Mean field games (MFG) is a recent theory invented by Lasry and Lions which studies optimization problems with large populations of agents in a dynamical framework. The mathematical analysis of such problems has so far focused on situations where the agents can evolve in the whole space. In practice, however, the agents often have constraints on their state. The aim of this Thesis is to understand the consequence of such constraints on the analysis of mean field games. We first show that the Nash MFG equilibria can be described as fixed points on the space of measures on constrained trajectories (generalized MFG equilibria). In order to obtain more precise results on these equilibria, we show a smooth optimality principle for the optimal trajectories of control problem with state constraints. We derive from this that the generalized equilibria satisfy a MFG system in which the Hamilton-Jacobi equation and the continuity equation have to be understand in a specific sense.
Mean Field Games Now Publishers
To represent the interaction of N rational competitors traditionally, a coupled system of N differential equations must be solved simultaneously, yielding the equilibrium strategy for each player. This approach becomes impractical as N grows, prompting the adoption of a mean-field approach, in which we assume N is large enough that the dynamics of the competition may be suitably represented distributionally in the continuum case. The trajectory of each player through the state space is then driven by a dynamic control, together with an adapted Brownian motion, reducing the computation from N differential equations to two Partial Differetial Equations (PDE's), a Hamilton-Jacobi-Bellman equation governing the evolution in time of a

utility function, and a Fokker-Planck (Forward Kolmogorov) equation governing the evolutio n in time of the distribution of players, coupled by means of the nonlinear Hamiltonian. In this thesis, we use the Implicit Function Theorem and Bifurcation Theory to obtain nontrivial equilibria of various mean-field games, and we go on to demonstrate the use of Schauder's fixed-point theorem to prove the existence of low-regularity time-bound solutions of a congestion-type mean-field game.

Probabilistic Theory of Mean Field Games with Applications I SIAM
This contributed volume considers recent advances in dynamic games and their applications, based on presentations given at the 17th Symposium of the International Society of Dynamic Games, held July 12-15, 2016, in Urbino, Italy. Written by experts in their respective disciplines, these papers cover various aspects of dynamic game theory including mean-field games, stochastic and pursuit-evasion games, and computational methods for dynamic games. Topics covered include Pedestrian flow in crowded environments Models for climate change negotiations Nash Equilibria for dynamic games involving Volterra integral equations Differential games in healthcare markets Linear-quadratic Gaussian dynamic games Aircraft control in wind shear conditions Advances in Dynamic and Mean-Field Games presents state-of-the-art research in a wide spectrum of areas. As such, it serves as a testament to the continued vitality and growth of the field of dynamic games and their applications. It will be of interest to an interdisciplinary audience of researchers, practitioners, and graduate students.

The Master Equation and the Convergence Problem in Mean Field Games Courier Corporation
This book describes the latest advances in the theory of mean field games, which are optimal control problems with a continuum of players, each of them interacting with the whole statistical distribution of a population. While it originated in economics, this theory now has applications in areas as diverse as mathematical finance, crowd phenomena, epidemiology, and cybersecurity. Because mean field games concern the interactions of infinitely many players in an optimal control framework, one expects them to appear as the limit for Nash equilibria of differential games with finitely many players as the number of players tends to infinity. This book rigorously establishes this convergence, which has been an open problem until now. The limit of the system associated with differential games with finitely many players is described by the so-called master equation, a nonlocal transport equation in the space of measures. After defining a suitable notion of differentiability in the space of measures, the authors provide a complete self-contained analysis of the master equation. Their analysis includes the case of common noise problems in which all the players are affected by a common Brownian motion. They then go on to explain how to use the master equation to prove the mean field limit. This groundbreaking book presents two important new results in mean field games that contribute to a unified theoretical framework for this exciting and fast-developing area of mathematics.

Mean-Field-Type Games for Engineers Springer Nature
This book provides a comprehensive introduction of Fog Radio Access Networks (F-RANs), from both academic and industry perspectives. The authors first introduce the network architecture and the frameworks of network management and resource allocation for F-RANs. They then discuss the recent academic research achievements of F-RANs, such as the analytical results of theoretical performance limits and optimization theory-based resource allocation techniques. Meanwhile, they discuss the application and implementations of F-RANs, including the latest standardization procedure, and the prototype and test bed design. The book is concluded by summarizing the existing open issues and future trends of F-RANs. Includes the latest theoretical and technological research achievements of F-RANs, also discussing existing open issues and future trends of F-RANs toward 6G from an interdisciplinary perspective; Provides commonly-used tools for research and development of F-RANs such as open resource projects for implementing prototypes and test beds; Includes examples of prototype and test bed design and gives tools to evaluate the performance of F-RANs in simulations and experimental circumstances.

Active Particles, Volume 1 Princeton University Press
The Encyclopedia of Systems and Control collects a broad range of short expository articles that describe the current state of the art in the central topics of control and systems engineering as well as in many of the related fields in which control is an enabling technology. The editors have assembled the most comprehensive reference possible, and this has been greatly facilitated by the publisher’s commitment continuously to publish updates to the articles as they become available in the future. Although control engineering is now a mature discipline, it remains an area in which there is a great deal of research activity, and as new developments in both theory and applications become available, they will be included in the online version of the encyclopedia. A carefully chosen team of leading authorities in the field has written the well over 250 articles that comprise the work. The topics range from basic principles of feedback in servomechanisms to advanced topics such as the control of Boolean networks and evolutionary game theory. Because the content has been selected to reflect both foundational importance as well as subjects that are of current interest to the research and practitioner communities, a broad readership that includes students, application engineers, and research scientists will find material that is of interest.

Mean Field Games with Heterogeneous Players Birkhäuser
A Course in Game Theory presents the main ideas of game theory at a level suitable for graduate students and advanced undergraduates, emphasizing the theory's foundations and interpretations of its basic concepts. The authors provide precise definitions and full proofs of results, sacrificing generalities and limiting the scope of the material in order to do so. The text is organized in four parts: strategic games, extensive games with perfect information, extensive games with imperfect information, and coalitional games. It includes over 100 exercises.

Paris-Princeton Lectures on Mathematical Finance 2010 Birkhäuser
This paper studies multidimensional mean field games with common noise and the related system of McKean-Vlasov forward-backward stochastic differential equations deriving from the stochastic maximum principle. We first propose some structural conditions which are related to the submodularity of the underlying mean field game and are a sort of opposite version of the well known Lasry-Lions monotonicity. By reformulating the representative player minimization problem via the stochastic maximum principle, the submodularity conditions allow to prove comparison principles for the forward-backward system, which correspond to the monotonicity of the best reply map. Building on this property, existence of strong solutions is shown via Tarski's fixed point theorem, both for the mean field game and for the related McKean-Vlasov forward-backward system. In both cases, the set of solutions enjoys a lattice structure, with minimal and maximal solutions which can be constructed by iterating the best reply map or via the fictitious play algorithm.

Game Theory with Engineering Applications Springer
This is the first tutorial to give such a concise and accessible introduction to game theory. It will be of use to all students,

practitioners, and researchers looking to understand the basic concepts, models, and applications.

Encyclopedia of Systems and Control Springer

The Paris-Princeton Lectures in Financial Mathematics, of which this is the fourth volume, publish cutting-edge research in self-contained, expository articles from outstanding specialists - established or on the rise! The aim is to produce a series of articles that can serve as an introductory reference source for research in the field. The articles are the result of frequent exchanges between the finance and financial mathematics groups in Paris and Princeton. The present volume sets standards with five articles by: 1. Areski Cousin, Monique Jeanblanc and Jean-Paul Laurent, 2. Stéphane Crépey, 3. Olivier Guéant, Jean-Michel Lasry and Pierre-Louis Lions, 4. David Hobson and 5. Peter Tankov.

Mean-Field Games MIT Press

This volume covers selected topics addressed and discussed during the workshop “PDE models for multi-agent phenomena,” which was held in Rome, Italy, from November 28th to December 2nd, 2016. The content mainly focuses on kinetic equations and mean field games, which provide a solid framework for the description of multi-agent phenomena. The book includes original contributions on the theoretical and numerical study of the MFG system: the uniqueness issue and finite difference methods for the MFG system, MFG with state constraints, and application of MFG to market competition. The book also presents new contributions on the analysis and numerical approximation of the Fokker-Planck-Kolmogorov equations, the isotropic Landau model, the dynamical approach to the quantization problem and the asymptotic methods for fully nonlinear elliptic equations. Chiefly intended for researchers interested in the mathematical modeling of collective phenomena, the book provides an essential overview of recent advances in the field and outlines future research directions.

Bertrand & Cournot Mean Field Games Springer Nature

This paper looks at a general framework for mean-field games with ambiguity averse players based on the probabilistic framework described in Carmona (2013). A framework for mean-field games with ambiguity averse players is presented, using a version of the stochastic maximum principle to find the optimal controls of the players. The dynamics under the optimal control are characterized through a forwards-backwards stochastic differential equation and a relationship between the finite player game and the mean-field game is established. Explicit solutions are derived in the case of the linear-quadratic mean-field game.

Differential Games Springer

Motivated by economical and engineering topics, around 2006, mean field games were introduced by Jean-Michel Lasry and Pierre-Louis Lions, and Peter E.~Caines, Minyi Huang and Roland P.~Malhamé, independently. This thesis addresses some mean field games models with free final time.In the first chapter, we consider several interacting populations evolving in R aiming at reaching given target sets in minimal time. The control system satisfied by each agent depends on an agent's position, the distribution of other agents in the same population, and the distribution of agents on other populations. Thus, interactions between agents occur through their dynamics. We consider in this chapter the existence of Lagrangian equilibria to this mean field game, their asymptotic behavior, and their characterization as solutions of a mean field game system, under few regularity assumptions on agents' dynamics. In particular, the mean field game system is established without relying on semiconcavity properties of the value function.Similarly to the first chapter, in the second chapter, we consider a mean field game model inspired by crowd motion where agents aim to reach a closed set, called target set, in minimal time, however in addition to congestion phenomena, which affects the velocity of an agent, the model is considered in the presence of state constraints: roughly speaking, these constraints may model walls, columns, fences, hedges, or other kinds of obstacles at the boundary of the domain which agents cannot cross. We first recall some previous results on the existence of equilibria for such games and presents the main difficulties that arise due to the presence of state constraints. Our main contribution is to show that equilibria of the game satisfy a mean field game system, thanks to recent techniques to characterize optimal controls in the presence of state constraints. These techniques not only allow to deal with state constraints but also require very few regularity assumptions on the dynamics of the agents.In our last chapter, we consider a mean field game model for crowd motion in which pedestrians interact not only through their position, but also through their velocity. More precisely, each pedestrian is assumed to minimize a cost involving their time to reach a certain target set, an individual integral cost, and an interaction integral cost modelling the fact that agents want to avoid congestion and prefer to move together with agents going in the same direction, in which can be seen as a Cucker--Smale type interaction. The main result we obtain in this chapter is the existence of equilibria for such a game, which is based on a variational approach.